Early Math Initiative

Shapes and Spatial Reasoning: The Development of Geometry Knowledge from Infancy Through the Early School Years

From birth, humans are aware of the objects in their environment. At only a few weeks old, infants can direct their gaze to objects and even track objects moving in space.[1–3] Through exploration and manipulation of objects, infants and children learn about the different attributes of objects such as their color, shape, or size. Infants also learn how objects move in space and interact with the environment. In the second year of life, as children's vocabulary grows, they learn words to describe geometric shapes (e.g., "Look, I have two circles.") or spatial directions (e.g., "Pick me up."). They mentally organize the world by classifying objects based on features, such as shape, and they learn to use their mental rotation skills to predict how objects will behave in space. Altogether, these skills are the foundation for children's developing understanding of geometry and play an important role in children's later development of mathematical knowledge.[4–6]

Geometry is a domain of math that involves the study of shapes and spatial reasoning. In early childhood, reasoning about shapes includes children's ability to tell the difference between various two- and three-dimensional shapes, match identical shapes, or classify shapes varying in size or color. Young children also learn the correct names of shapes, in English or their home language, and begin to learn about the attributes of individual shapes (e.g., squares have four equal sides and four right angles).

Young children learn the names of shapes, in English or their home language, and begin to learn about the attributes of individual shapes.

In This Brief

- A summary of research findings on the development of geometry skills and concepts from infancy through the early school years
- Practical implications for educators and caregivers working with children on how they can support children's development of geometry skills and concepts through everyday interactions and learning activities

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Spatial reasoning (also referred to as spatial sense) is composed of various components. It involves spatial orientation (knowing where you or objects are in space, e.g., Are you close to or far away from the table?), spatial navigation (knowing how to navigate from one place to another using mental representations of the environment, e.g., How can you get from the chair to your toy?), and spatial visualization (imagining what objects look like when turned upside down or rotated, e.g., mental rotation).

Spatial skills support lifelong learning in the areas of science, technology, engineering, and mathematics.

Spatial skills can be practiced and developed with experience. Numerous studies have shown improvement in children's and adult's performance on spatial tasks with practice or training.[7,8] Although, on the surface, spatial skills such as mental rotation may not seem directly related to mathematical learning, research suggests that spatial skills support lifelong learning in the areas of science, technology, engineering, and mathematics.[4–6] For example, learning how to create patterns with shapes (e.g., circle, circle, square, circle, circle, square) helps children understand foundational elements of algebra. When children think about the positions of objects in space and their distance from one another, they learn about basic concepts of measurement. Furthermore, research suggests that early mental rotation skills may support children's solving of arithmetic problems.[9,10]

This brief aims to present an overview of children's developing geometry skills and concepts, including spatial reasoning and understanding of shapes. It also provides practical implications of how educators can best support these skills in early childhood.

A summary of foundations and standards in geometry for infants and toddlers, preschoolers, and early elementary school students in California is presented in Appendix A.

Developing the Foundations of Geometry in Infancy

Infants are born with the ability to develop an understanding of the spatial world that they live in. From a very young age, infants attend to similarities and differences between various objects and people. As their motor skills develop and they gain experience interacting with various objects, infants develop their spatial orientation and begin to recognize a few basic shapes.

Object Perception

Infants are surrounded by objects from the moment they are born. Through experiences and explorations of objects, infants discover how objects move and behave in space. For example, they may notice that when banging two toys, the toys do not fuse together. Instead, each toy remains a separate object and maintains its connectedness and boundaries (principle of cohesion). They also learn that one object cannot pass through another (principle of solidity) and that objects do not pop in and out of existence even when the object is briefly invisible. Lastly, they expect a moving object that becomes hidden from view to move along one continuous path through space (principle of continuity). For example, when a ball rolls behind the couch, infants expect the ball to continue rolling and reappear on the other side.[11–13] By four or five months of age, infants already have a strong understanding of these core principles and can predict when and how objects will move in space. Such knowledge guides infants' interactions with new objects.[12,14,15]

Infant
- Follows objects moving in space
- Differentiates between basic shapes (e.g., circle and square)
- Notices locations and positions of objects in space
- Begins to use landmarks to navigate space

Toddler
- Identifies basic shapes (e.g., circle, square, equilateral triangle)
- Distinguishes between shapes based on perceptual features
- Develops greater motor skills which results in improved spatial navigation
- Understands limited spatial vocabulary (e.g., in, on, under)

Preschool
- Classifies shapes by perceptual features
- Composes new shapes from other shapes (e.g., two triangles make a square)
- Rotates objects mentally
- Develops spatial vocabulary (e.g., front, inside, between)
- Begins to create a coherent mental map of the surrounding space

Elementary
- Classifies shapes by defining attributes
- Recognizes many 2D and 3D shapes
- Composes and decomposes shapes from other shapes (e.g., decomposes a hexagon into six triangles)
- Solves complex mental rotation problems that, for example, involve abstract objects or several rotations.

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As infants learn about the principles that guide object movement, they also gain an understanding of the attributes of objects. They learn about an object's size, shape, color, or texture. Knowledge of these attributes allows children to organize the world around them and to understand how objects may interact with one another. For example, research suggests that four-month-old infants know that a wide object cannot fit inside a narrow container.[16] By eight months of age, infants even learn to adapt grasping behavior to match the size of the object they are grasping. For example, eight-month-olds will extend both hands when reaching toward a large object, but will only extend one hand when reaching for a small object.[17] By 12 months of age, infants understand that the shape of an object can affect how it moves. For example, they are surprised when they see a cube roll but not when they see a ball roll.[18]

Infants' initial knowledge of physical principles and their growing understanding of objects' attributes form the basis for their developing geometry concepts.

Shape Recognition

Infants are very attentive to shapes in their environment. Research suggests that by three to four months of age infants not only notice the difference between basic shapes such as circles, squares, equilateral triangles, and crosses but also have formed mental categories for these shapes.[19] In one study, after infants had been presented with multiple examples of the same shape (e.g., six different squares), they were shown a new shape of the same shape category (e.g., a new square) and a new shape of a different shape category (e.g., a circle). The researchers found that infants three to four months old, but not newborns, looked significantly longer at the novel shape category (e.g., the circle), which suggests that after only a few months of experience with shapes in their environment, infants have formed categories for some basic shapes.[19]

Infants' spatial reasoning is developed through their physical manipulation of objects in their environment.

In the first two years of life, though, shape recognition is still very limited. Infants will recognize and match only basic shapes such as circles and squares (and occasionally triangles). Furthermore, they will only consider shapes to be the same if the shapes are also identical in size and orientation. For example, a square presented on its side and another presented with the point down (in a diamond orientation) are viewed as two different shapes.[20]

Mental Rotation Skills

Infants' spatial reasoning is developed through their physical manipulation of objects in their environment and is tied to their developing motor skills. By holding and moving objects, infants learn not only about the attributes of those objects (e.g., the toy giraffe has a round shape on top, the head, and long shapes on the bottom, the legs) but also how manipulating objects changes what the objects look like. For example, they learn that a toy giraffe looks different when turned around or upside down.

Such physical manipulations of objects help infants develop their mental rotation skills. Mental rotation is the ability to imagine what an object might look like when rotated. One way researchers have investigated the development of infants' mental rotation skills is by studying their ability to discriminate between an object and its mirror image. In this type of study, researchers place an asymmetrical object, such as the letter P, behind a screen. When the screen is removed, one of two things is shown: the same object, but rotated, or a mirror image of the original object.[21,22] Infants' longer gaze at the mirror image than at the rotated original object suggests that they recognize that the mirror image is a different object and that the rotated original is the same object.

One study showed that infants who were allowed to physically manipulate the object beforehand were able to discriminate mirror images as early as six months of age.[22] In other words, infants' physical experience with an object helped them think

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about that object in space and recognize that when the object is rotated, it is still the same object. Another study found that infants 10 months of age (but not infants 8 and 9 months of age) were able to discriminate between objects that are mirror images after observing the objects being manipulated by someone else as opposed to physically interacting with the objects themselves.[21] Infants' motor development also plays a role in their ability to think spatially; in the same study just described, infants who were further along in their gross motor development were better able to discriminate between mirror images.

Navigation and Orientation

Another important component of spatial thinking involves spatial orientation, or understanding relationships between positions in space (knowing where objects are in relation to one's own position or in relation to another object), and spatial navigation, or how to navigate the environment (knowing how to get from point A to point B). Infants can represent the position of objects in space and rely on salient cues in the environment to record the location of an object.[23]

To study infants' early navigation and orientation abilities, researchers have used a type of peekaboo game. In this game, an infant is placed in a room and is trained to expect a person to appear from the same window on one side of the room (e.g., the left) every time they hear a sound. The infant is then moved to the opposite side of the room so that their view of the room is now reversed. When hearing the sound in this new orientation, infants tend to anticipate the appearance of the person by continuing to look to the left, despite the fact that in this new orientation the person appears to the right.[24,25]

Learning the names for shapes can play a very important role in toddlers' ability to recognize and match a greater variety of shapes.

However, by about eight months of age, infants become more aware of their surroundings and start using important landmarks in their environment (e.g., the location of a light or painting in a room) to orient themselves. Researchers found that when infants were tested on a version of the peekaboo game that had clear landmarks (e.g., lanterns hanging next to the window where the person appeared), eight-month-old infants, but not six-month-old infants, looked to the correct window after they were moved to the new, reversed location.[26]

Reasoning About Shape and Space in Toddlerhood

As toddlers' language skills rapidly develop, so does their language to describe shape and space. This development not only allows toddlers to talk about the world using shape and spatial language, but it also helps them build stronger spatial reasoning skills.

Learning to Identify Shapes

Two-year-olds are able to recognize and name a few basic shapes such as circles, squares, and occasionally triangles.[27] However, at this age, toddlers' ability to recognize shapes is highly dependent on how similar they are to the typical version of that shape. That is why toddlers find it fairly easy to recognize circles, but they have a harder time recognizing triangles. Although circles can vary in size, they look the same even when rotated. However, other shapes like triangles have a lot more variation. Not only can triangles be of different sizes and orientations, but the angles and lengths of the sides can also change. Toddlers, therefore, are generally limited to recognizing equilateral triangles (e.g., the "typical triangle") but fail to recognize other, less typical, types of triangles (e.g., isosceles or right-angle triangles).

Toddlers' shape categorization is based in large part on the general perceptual features of the shape (e.g., round or pointy,

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point at the top or bottom) rather than the shape's more specific attributes (e.g., the number and length of the sides, the angles). As such, toddlers often make errors when naming shapes. For example, toddlers will often say that ovals are circles and that rectangles are squares.[20] Even so, research suggests that learning the names for shapes can play a very important role in toddlers' ability to recognize and match a greater variety of shapes.[28] One study found that between the ages of 25 and 30 months, toddlers are very quickly acquiring new shape names, which allows them to recognize a greater variety of shapes at 30 months than at 25 months. Furthermore, 30-month-olds were able to recognize less typical versions of shapes (e.g., triangles that are not equilateral).

Motor Development Supports Spatial Thinking

As their motor skills develop, toddlers undergo major development in their spatial navigation and mental rotation skills, specifically between the ages of 15 and 30 months.[29] As toddlers learn to walk and gain experience viewing the environment from different perspectives, they begin to notice and use landmarks as a way to navigate space.[30] Whereas infants and younger toddlers notice when something has been rotated,[31] older toddlers begin to develop the ability to mentally rotate an object to reach a goal, such as fitting the object into a hole. For instance, 22-month-olds can rotate a block to the correct orientation to put it in the correct hole of a shape-sorter box.[32]

Spatial Vocabulary

Spatial vocabulary refers to words that describe the position, location, or movement of objects or people. Spatial vocabulary can be divided into three broad categories of words:

1. Position or location: on, in, over, under, behind, near, between
2. Direction: up, down, left, right, across, upside down
3. Distance: near, far, long, farther, away

Providing an environment that is rich with spatial vocabulary, in children's home languages or in English, can draw children's attention to the location, position, and direction of objects in space. This practice is particularly important as adults' and children's use of spatial vocabulary in the first few years of life predicts children's spatial problem solving later on.[33]

Learning Spatial Vocabulary

Toddlers also develop vocabulary for describing spatial positions, locations, directions, or distance (see the box above about spatial language).[33,34] For example, they might point out that their ball is "under" the table or ask to go "down" the slide. Researchers hypothesize that building spatial vocabulary helps young children think about the world spatially and build stronger spatial reasoning skills. For example, toddlers between the ages of 16 and 24 months who had a vocabulary that included prepositions such as "up," "here," and "out" were better able to learn the location of a hidden toy than those who did not have this vocabulary.[34]

In another study, children between the ages of 14 and 46 months who spontaneously used more spatial vocabulary (e.g., long, triangle, end) were later more skilled at nonverbal spatial problem-solving tasks at the age of 54 months.[33] For example, children who used more spatial words were more skilled at identifying the shape that would result from rotating and combining two smaller shapes. They were also more skilled at a spatial analogy task in which they chose an image that showed the same spatial relationship as a target image (e.g., comparing a picture of a bird flying above a tree to a picture of a dot above a circle, and recognizing that the spatial relationship between the bird and the tree is the same as that of the dot and the circle).

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Practical Implications for Adults Working with Infants and Toddlers

Infants' and toddlers' growing knowledge of shape and spatial reasoning skills can be supported by using spatial words and shape names during daily routines and play. This practice will help children notice that shapes are all around them and will help them build spatial concepts and skills that are important for their later mathematical learning.

Research-Based Strategies

- Place interesting objects in the child's visual field, alongside the head or from above (such as a mobile), to allow the child to track movement and to reach out for objects.
- Provide infants with mobiles or other objects hanging from the ceiling for them to track with their eyes or to reach for and move. This practice will help them learn how objects can move in space.
- Create many experiences for infants and toddlers to physically manipulate objects. This exploration will help them learn about the characteristics of objects and how objects varying in size and shape fit in space. By manipulating objects, children also explore how rotation transforms objects.[31]
- Highlight spatial language (e.g., on, into, under, in, above, up, down, between) during daily routines. For example, "I'm putting a new diaper on your body," "Let's climb up the step ladder to wash our hands," or "I'm pouring the milk out of the carton into your sippy cup."
- Support the use of spatial language in the children's home language or in English to promote their spatial reasoning and language development.[33] Once children learn a particular concept, this knowledge transfers across languages. For example, a toddler may learn the concept of "under" from interactions at home in Spanish and may then transfer this knowledge when learning the word "under" in English.
- Provide children opportunities to sort shapes by category (e.g., put all circles in one basket and all squares in another basket). Use language describing shapes (in the child's home language or in English) to explain why certain shapes belong to certain categories (e.g., "This shape is round, and this shape is also round. They are both circles.").
- Encourage children to play with blocks varying in size and shape from an early age. Allow children to practice stacking blocks to build basic spatial skills.
- Play shape-matching games. Provide children with shapes such as circles and squares of various colors and ask them to find a match to each shape. As children become more confident with this task, provide them with shapes that vary in size as well as color.
- Create an obstacle course for children to explore with different types of gross motor equipment like tunnels, ramps, or lofts. This exploration will give children the opportunity to see the world from different perspectives and learn how their body fits and moves in space.

Reasoning About Shape and Space in the Preschool Years

During the preschool years, children make important improvements in their geometry skills. Preschoolers are not only able to recognize a greater variety of shapes but are also beginning to pay attention to the attributes of shapes when identifying them. At the same time, they make important strides in their mental rotation skills and learn to use these skills to reach a goal, such as fitting a block in a tower they are building.

Learning About Shape Attributes

As children enter preschool they begin to recognize and name a wider variety of shapes, including rectangles and less typical triangles, with increasing accuracy. When presented with a set of typical and less typical shapes such as those shown in Figure 1, 92 percent of four-year-olds correctly identified the circles, and 82 percent correctly identified the squares. Although preschoolers are able to recognize a greater variety of triangles, including less typical triangles (e.g., isosceles triangles), still only 60 percent correctly identified all triangles.[35]

Like toddlers, preschoolers base their decisions on the general perceptual features of a shape.[20,35] For example, they might believe that having a point at the top is a typical feature of a triangle. Using this perceptual feature, preschoolers may reject the less typical triangles shown in Figure 1 because none have the point at the top but accept the nonvalid triangles because they do have the point at the top. Preschoolers are still developing their understanding of what attributes such as points or sides really mean.[35]

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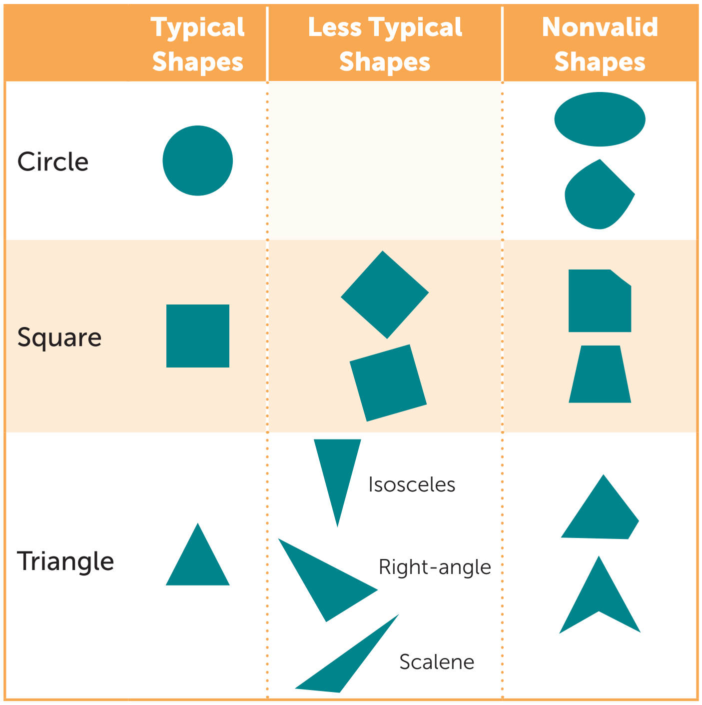

Figure 1, adapted from Satlow & Newcombe (1998)[36]

Additionally, children may focus on one important attribute of a shape but not others. For example, a preschooler may know rectangles have four sides but does not take into account that all angles in a rectangle measure 90 degrees. Consequently, preschoolers may identify rhombuses as being rectangles because they meet some but not all of the attribute requirements.[35] It is not until early elementary school that children will shift their decision-making process to prioritize the defining attributes of shapes over perceptual features.[36]

In playing with blocks, preschoolers also begin to compose shapes from other shapes. For example, preschoolers will stack differently shaped blocks to build houses, towers, or arches, sometimes even of multiple levels. However, this process still involves trial and error. Unlike older children who can anticipate what figures will be produced when combining several blocks together (e.g., four blocks connected at right angles will produce an enclosure), preschoolers rely on trial and error of many different configurations of blocks before finding a construction that works.[20]

Developing Mental Rotation Skills

During the preschool years, children's spatial reasoning continues to improve, and they are more skilled at mentally rotating objects than they were in toddlerhood.[37,38] For example, if shown a picture of an elephant that is slightly rotated so that it is not in a typical standing position, most four-year-olds can tell which direction (right or left) the elephant would be facing if it were rotated to be on its feet. This task becomes harder for preschoolers when the angle of rotation is larger (e.g., an elephant rotated upside down with its legs in the air).

Additionally, rotating three-dimensional objects is more challenging than rotating two-dimensional objects.[39,40]

Children's understanding of spatial vocabulary helps them solve spatial problems.

However, children's skill in mentally rotating two-dimensional objects can improve with training that involves rotating objects or gesturing about object rotations.[37,38] Between the ages of three and five, children become much more efficient in mentally rotating objects to reach a goal, such as fitting a puzzle piece into a hole without having to rely on physical trial and error.[41]

During this period, children can also reason by analogy about spatial relationships. For example, many preschoolers can solve the following spatial analogy just as well as adults can, showing that they are able to reason about the relationships between the parts of the human body and map those relationships to a completely different image: "If this mountain had a knee, where would it be?"[42] Additionally, children's understanding of spatial vocabulary continues to develop during the preschool years, which helps them solve spatial problems. For example, one study found that preschool-age children who know the words "middle" and "between" were better able to find an object hidden at the midpoint between two landmarks than children who did not know those words.[43]

Navigation and Orientation

As children's motor skills continue to refine and as they become more comfortable navigating their environment, they develop the ability to combine their different perspectives of a space into a coherent mental map.[30] For example, they can imagine what

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Practical Implications for Adults Working with Preschoolers

To support preschoolers' spatial reasoning skills and their understanding of shapes, provide them with varied experiences to physically explore and manipulate a variety of two- and three-dimensional shapes. Such experiences can help children build spatial and shape vocabulary as well as their understanding of spatial relationships and the various attributes of shapes.

Research-Based Strategies

- In daily routines and interactions, invite children to identify circles, squares, rectangles, and triangles in their environment, in the language in which they feel most comfortable. For example, invite children to identify shapes in the environment by playing "I Spy."
- Provide varied examples of each shape category. For example, when talking about triangles, include typical and less typical triangles such as equilateral, isosceles, right-angled, and scalene triangles.[20]
- Invite children to match, sort, and classify shapes. As children identify various shapes, discuss the attributes relevant to the shape (e.g., number of sides, angles). For example, ask children to identify how many sides and corners are in a rectangle.
- Play with puzzles.[44] Puzzle play allows children to practice mentally and physically rotating puzzle pieces into the correct location and to visualize how the pieces may fit together. This type of play is also a great opportunity for children and adults to practice using spatial language when talking about the position and direction of different puzzle pieces (e.g., "this corner piece goes at the top" or "I think that piece is upside down").
- Provide children with opportunities to explore and represent shapes in a variety of ways. For example, hide a shape in a bag and ask children to feel the shape, describe what they notice (e.g., "it is pointy"), and identify the shape. Invite children to create shapes using different materials (e.g., playdough, pipe cleaners, straws).
- Encourage children to play with blocks. Block play allows children to practice mental manipulations of objects and to predict the effects of transformations (rotation, flipping, sliding) on various shapes. As children become more confident in building simple structures, encourage them to create more complex structures using various shapes. Block play has been shown to improve children's spatial skills in kindergarten and beyond.[6,45]
- Use gestures when explaining spatial transformations or movements and encourage children to do the same. This practice may be especially beneficial for dual language learners to develop an understanding of spatial concepts and the English words to describe them. For example, when explaining how a block needs to be rotated to fit on a block tower, use gestures as well as spatial language to demonstrate the rotation. Children will have to use mental rotation and visualization to imagine how the block needs to be rotated.[38,46]

the room looks like from the perspective of someone walking through the door or from someone standing outside the window looking in. They may use multiple landmarks to find an object (e.g., my toy was under the window, next to the ball) and begin thinking about distance (e.g., my toy is close to the table but far from the lamp).[30,34]

More Advanced Shape Concepts and Spatial Thinking Skills in the Early Elementary School Years

As children enter early elementary school, they understand the importance of shape attributes in identifying and classifying shapes. They begin to use attributes, such as the number or lengths of sides to recognize more complicated two- and three-dimensional shapes. At the same time, children of this age are able to use their mental rotation skills to rotate more complicated shapes and use these newly developed skills during play, such as when building with blocks or solving puzzles.

Shape Composition and Decomposition

By the time children reach kindergarten they are able to recognize more complicated shapes such as hexagons, rhombuses, and trapezoids.[20] They also learn to identify some three-dimensional shapes, such as cubes, cones, cylinders, and spheres, and can begin to compare and contrast two- and three-dimensional shapes. This period is also the time at which children are more likely to attend to the defining attributes of shapes when identifying and classifying shapes. Children now understand that an object can only be classified as a particular shape if it has all the defining attributes of that shape. For example, a rectangle not only has four sides, but it also has four

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right angles. Now that children are paying more attention to object attributes, they are also learning about angles. In the early elementary years, children can recognize right angles and understand that angles are critical parts of shapes.[20]

Children also become more proficient at composing and decomposing various shapes. Children in early elementary school can make pictures out of two-dimensional shapes. At the same time children begin to decompose shapes into smaller shapes (a hexagon into six triangles) and begin to find "hidden shapes" within a complex diagram or picture. Using three-dimensional shapes such as blocks, children are able to make complex structures, such as towers, arches, or bridges, or make buildings with multiple levels and ceilings.[20]

Mental Rotation Skills Predict Later Math Skills

During the kindergarten and early elementary school years, children continue to develop their spatial thinking ability. For example, they become skilled at more challenging mental rotation problems, including those involving more abstract items (e.g., mentally rotating letters) and the rotation of three-dimensional objects (e.g., a cube).[47] Research suggests that mental rotation skills predict later performance on arithmetic skills. One study found, for example, that children's mental rotation skills in kindergarten predict how well they can solve arithmetic problems in second grade.[10]

Although researchers are still trying to understand why children's early spatial skills are predictive of their later arithmetic skills, one hypothesis is that early spatial skills help children think about numbers in a more visual way, such as a number line with small numbers on the left and large numbers on the right. Thinking about numbers in this visual way is important because it helps children think about the relationships between numbers and how big or small they are (e.g., four is to the left of five, which means four is smaller than five). This skill is important when solving arithmetic problems.[48–51]

Practical Implications for Adults Working with Early Elementary School Students

Children's growing understanding of shapes and space allow for more advanced spatial and shape problem-solving activities. The development of spatial skills is important for their later mathematical development.[4,5,52]

Research-Based Strategies

- Continue to provide many opportunities for block building, including structured and unstructured block play. For structured block play, give children a specific block problem to build (e.g., an enclosure with four walls and a ceiling) so that they can work on their spatial problem solving.[52] At this age, children are able to engage in block play more independently and build increasingly more complex structures.
- Encourage children to play with puzzles. Use spatial language to describe where a puzzle piece may go (e.g., up, left, right, side) in English or the child's home language.[44]
- Use gestures to describe and point out various attributes of shapes (e.g., sides and corners) and the relationships between the attributes (e.g., whether two sides are parallel or perpendicular). This scaffolding supports dual language learners to learn about the attributes as well as the English words associated with them.
- Challenge children to construct pictures from various shapes (e.g., blocks). For example, ask them to make a flower using only two types of shapes.
- Invite children to superimpose tangrams on pictures in which tangram shapes have been illustrated (e.g., a Christmas tree made of three triangles and a rectangle). Encourage children to flip or rotate the tangram pieces until they fit.
- Use spatial language during everyday activities and interactions, especially during those instances that require spatial problem solving. For example, if children struggle to fit all the letters of a word onto a page when writing, talk about how much space (e.g., small or big) they should leave so that all the letters can fit. Or talk about how children can reorient or turn objects (e.g., on top, to the left, turn it around) so that they best fit in their desks.

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Conclusion

From the moment infants are born, they track the objects around them and develop an understanding of the principles that guide how their own bodies and objects move in space. Through increased physical exploration and experimentation with objects, infants and toddlers learn to mentally rotate shapes and imagine what they may look like from different perspectives. As toddlers learn to crawl and walk, they undergo major development in their navigation skills and begin to use landmarks as a way to navigate space. Simultaneously, toddlers are rapidly learning more spatial and shape vocabulary, which helps them build stronger geometry and spatial reasoning skills. Highlighting spatial vocabulary in children's home language or in English during daily routines and activities is therefore very important.

As children enter preschool, they begin to compose and decompose shapes using materials like blocks or tangrams. However, it is not until the early elementary school years that children understand the importance of shape attributes (e.g., the length of sides or the type of angles) in classifying shapes. Through play with blocks and puzzles, children in preschool and the early elementary years continue to develop their mental rotation skills, learning to tackle more complex problems including the rotation of three-dimensional objects. Providing children with many opportunities to physically explore the objects in their environment is key to their development of the geometry skills that provide the foundation for later mathematical learning.[4–6]

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Appendix A: California Early Learning Foundations and Standards in Geometry

California Infant-Toddler Learning Foundations

Foundation: Spatial Relationships

The developing understanding of how things move and fit in space

8 months	18 months	36 months
At around eight months of age, children move their bodies, explore the size and shape of objects, and observe people and objects as they move through space.	At around 18 months of age, children use trial and error to discover how things move and fit in space. (12–18 mos.; Parks 2004, 81)[53]	At around 36 months of age, children can predict how things will fit and move in space without having to try out every possible solution, and show understanding of words used to describe size and locations in space.

Source: California Infant-Toddler Learning Foundations[54]

California Preschool Learning Foundations in Mathematics

Geometry: Substrand 1.0 and 2.0

	At around 48 months of age	At around 60 months of age
Geometry	**1.0 Children begin to identify and use common shapes in their everyday environment.** 1.1 Identify simple two-dimensional shapes, such as a circle and square. 1.2 Use individual shapes to represent different elements of a picture or design.	**1.0 Children identify and use a variety of shapes in their everyday environment.** 1.1 Identify, describe, and construct a variety of different shapes, including variations of a circle, triangle, rectangle, square, and other shapes. 1.2 Combine different shapes to create a picture or design.
Geometry	**2.0 Children begin to understand positions in space.** 2.1 Identify positions of objects and people in space, such as in/on/under, up/down, and inside/outside.	**2.0 Children expand their understanding of positions in space.** 2.1 Identify positions of objects and people in space, including in/on/under, up/down, inside/outside, beside/between, and in front/behind.

Source: California Preschool Learning Foundations in Mathematics[55]

The California Statewide Early Math Initiative is funded by
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California Common Core State Standards for Mathematics: Kindergarten–Grade 2

Domain: Geometry

	Kindergarten	Grade 1	Grade 2
Geometry	K.G	1.G	2.G
	Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	**Reason with shapes and their attributes.**	**Reason with shapes and their attributes.**
	1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to*. 2. Correctly name shapes regardless of their orientations or overall size. 3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").	1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. 2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[1] 3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths,* and *quarters*, and use the phrases *half of, fourth of,* and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. **Footnote:** 1. Students do not need to learn formal names such as "right rectangular prism."	1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[2] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. 2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. 3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. **Footnote:** 2. Sizes are compared directly or visually, not compared by measuring.
	Analyze, compare, create, and compose shapes.		
	4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). 5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. 6. Compose simple shapes to form larger shapes. For example, *"Can you join these two triangles with full sides touching to make a rectangle?"*		

Source: California Common Core State Standards for Mathematics[56]

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